# Appendix

## Supplementary material

### Mathematical formalism

**Theorem 1**. For a specific region $i$, the $CSIR$ on day $t$ will decrease if the proportion of new cases for this region on day $t$ will be not greater than the proportion of all cases for this region on day $t-1$.

*Proof:*

Let $i$ be a specific region and $CSIR_i(t-1) = O_{i,t-1}/E_{i,1:(t-1)}$ and $CSIR_i(t) = (O_{i,t-1} + x_{i,t})/E_{i,1:t}$ be the cumulative estimates of RRs for region $i$ on days $t-1$ and $t$, respectively, where $O_{i,t-1}$ denotes the total (historical) sum of observed number of cases for $i$ registered on day $t-1$, $x_{i,t}$ denotes new cases for $i$ confirmed on day $t$, and $E_{i,1:(t-1)}$ and $E_{i,1:t}$ denote the expected cumulative number of cases for $i$ on days $t-1$ and $t$, respectively. The decrease in $CSIR$ between days $t-1$ and $t$ may be written in the form of an inequality:

$$\frac{O_{i,t-1}+x_{i,t}}{E_{i,1:t}} \leq \frac{O_{i,t-1}}{E_{i,1:(t-1)}} \quad \leftrightarrow \tag{7}$$

$$O_{i,t-1} + x_{i,t} \leq \frac{O_{i,t-1}E_{i,1:t}}{E_{i,1:(t-1)}} \quad \leftrightarrow \tag{8}$$

$$x_{i,t} \leq \frac{O_{i,t-1}E_{i,1:t}}{E_{i,1:(t-1)}} - \frac{O_{i,t-1}E_{i,1:(t-1)}}{E_{i,1:(t-1)}} = \frac{O_{i,t-1}(E_{i,1:t}-E_{i,1:(t-1)})}{E_{i,1:(t-1)}} \quad \leftrightarrow \tag{9}$$

$$\frac{x_{i,t}}{O_{i,t}} \leq \frac{E_{i,1:t}-E_{i,1:(t-1)}}{E_{i,1:(t-1)}} \quad \leftrightarrow \tag{10}$$

But $E_{i,1:(t-1)} = P_i r_{t-1}$ and $E_{i,1:t} = P_i r_t$ with $r_{t-1} = \frac{O(t-1)}{\sum_i P_i}$ and $r_t = \frac{O(t-1)+x(t)}{\sum_i P_i}$, where $O(t-1)$ denotes the total (historical) sum of observed number of cases for all regions on day $t-1$, $x(t) = \sum_i x_{i,t}$ denotes the sum of cases confirmed for all regions on day $t$, $P_i$ is the population size in $i$, and $\sum_i P_i$ is the country-wide population. Thus,

$$\frac{x_{i,t}}{O_{i,t-1}} \leq \frac{P_i(r_t - r_{t-1})}{P_i r_{t-1}} = \frac{r_t - r_{t-1}}{r_{t-1}} \tag{11}$$

Substituting for $r_t$ and $r_{t-1}$:

$$\frac{x_{i,t}}{O_{i,t-1}} \leq \frac{\frac{O(t-1)+x(t)}{\sum_i P_i} - \frac{O(t-1)}{\sum_i P_i}}{\frac{O(t-1)}{\sum_i P_i}} = \frac{\frac{x(t)}{\sum_i P_i}}{\frac{O(t-1)}{\sum_i P_i}} = \frac{x(t)}{O(t-1)} \tag{12}$$

But the above result is equivalent to:

$$\frac{x_{i,t}}{x(t)} \leq \frac{O_{i,t-1}}{O(t-1)} \tag{13}$$

Note that $\frac{x_{i,t}}{x(t)}$ is the proportion of cases for region $i$ on day t, while $\frac{O_{i,t-1}}{O(t-1)}$ is the 'cumulative' proportion of cases for region $i$ on day $t-1$, *i.e.* the total (historical sum) of confirmed cases for $i$ up to day $t-1$, divided by the total (historical sum) of confirmed cases in the country up to day $t-1$.

It is also possible to write a simplified version of the above proof. Let $O(t) = \sum_i O_{i,t}$, $P = \sum_i P_i$ , $\rho_i = \frac{P_i}{P}$ and $x(t) = \sum_i x_{i,t}$ . Then

$$CSIR_i(t) = \frac{O_{i,t}}{E_{i,t}} = \frac{O_{i,t}}{\frac{P_i O(t)}{P}} = \frac{1}{\rho_i}\frac{O_{i,t}}{O(t)}. \tag{14}$$

Then,

$$CSIR_i(t-1) \geq CSIR_i(t) \leftrightarrow \tag{15}$$

$$\frac{1}{\rho_i}\frac{O_{i,t-1}}{O(t-1)} \geq \frac{1}{\rho_i}\frac{O_{i,t}}{O(t)} \leftrightarrow \tag{16}$$

$$\frac{O_{i,t-1}}{O(t-1)} \geq \frac{O_{i,t}}{O(t)} \leftrightarrow \tag{17}$$

$$O_{i,t-1}O(t) \geq O_{i,t}O(t-1) \tag{18}$$

$$O_{i,t-1}\big(O(t-1)+x(t)\big) \geq (O_{i,t-1}+x_{i,t})O(t-1) \tag{19}$$

$$O_{i,t-1}x(t) \geq x_{i,t}O(t-1) \tag{20}$$

$$\frac{O_{i,t-1}}{O(t-1)} \geq \frac{x_{i,t}}{x(t)} \tag{21}$$

**Theorem 2.** The WCSIR will decrease if either of the two following conditions in the form of conjunctions are met (we note that an analogous theorem can be formulated for an increasing WCSIR):

(i) The CSIR is decreasing (proportion of infected for region $i$ on day $t$ is not greater than the cumulative proportion of infected for region $i$ on day $t-1$ ) and the CSTR is increasing (proportion of tests in region $i$ on day $t$ is not smaller than the cumulative proportion of tests on day $t-1$.)

(ii) LPR is decreasing (the rate of positive cases for region $i$ on day $t$ is not greater than the cumulative rate of positive cases on day $t-1$) and GPR is increasing (the rate of positive cases for the whole country on day $t$ is not smaller than the cumulative rate for the whole country of positive cases on day $t-1$).

*Proof:*

(i)

If $CSIR_i(t) \leq CSIR_i(t-1)$ and $CSTR_i(t) \geq CSTR_i(t-1)$, then (22)

$$WCSIR_i(t) = \frac{CSIR_i(t)}{CSTR_i(t)} \leq \frac{CSIR_i(t-1)}{CSTR_i(t-1)} = WCSIR_i(t-1). \tag{23}$$

(ii)

If $LPR_i(t) \leq LPR_i(t-1)$ and $GPR(t) \geq GPR(t-1)$, then (24)

$$WCSIR_i(t) = \frac{LPR_i(t)}{GPR(t)} \leq \frac{LPR_i(t-1)}{GPR(t-1)} = WCSIR_i(t-1). \tag{25}$$

**Data availability statement**

Processed data on infections and testing are available at https://github.com/michalmichalak997/COVID-19/blob/master/Data%20and%20code/Data%20and%20code_December_4_update.zip

Original data on infections and testing are available at https://twitter.com/mz_gov_pl?lang=pl

Processed data that are used for generating individual plots are available at https://github.com/michalmichalak997/COVID-19/blob/master/Data%20and%20code/Data%20for%20figures_December_4_update.zip

Population census data are available at https://stat.gov.pl/en/

**Code availability**

Computer code is available at https://github.com/michalmichalak997/COVID-19/blob/master/README.md.
Interactive plots are publicly available at https://michalmichalak997.shinyapps.io/shiny_corona/

**Supplementary tables and figures**

**Table S1. Reported inaccuracies related to the number of confirmed cases.**

| Error ID | No. of errors | Reporting date (2020) | Error date (2020) | Correction date (2020) | Reference |
|---|---|---|---|---|---|
| 1 | 4 | June 26 | ND | June 25 | [1] |
| 2 | 5 | June 26 | ND | June 24 | [2] |
| 3 | 51 | June 19 | ND | June 16-19 | [3] |
| 4 | 1 | June 2 | ND | June 1 | [4] |
| 5 | 2 | June 9 | June 6 | June 6 | [5] |
| 6 | 4 | May 31 | ND | May 31 | [6] |
| 7 | 1 | May 29 | ND | May 27 | [7] |
| 8 | 2 | May 26 | ND | May 26 | [8] |
| 9 | 1 | June 1 | ND | May 25 | [9] |
| 10 | 34 | May 25 | ND | May 25 | [10] |
| 11 | 4 | May 23 | ND | May 23 | [11] |
| 12 | 2 | May 25 | ND | May 23 | [12] |
| 13 | 39 | May 13 | May 12 | May 12 | [13] |
| 14 | 21 | May 8 | May 7 | May 7 | [14] |
| 15 | 17 | May 7 | May 5 | May 5 | [15] |
| 16 | 2 | May 6 | ND | May 5 | [16] |
| 17 | 63 | April 30 | April 16-19 | April 16-19 | [17] |
| 18 | 5 | June 2 | ND | June 1 | [18] |

**Table S2. Cumulative number of tests as of the 26th of March 2020.**

| Region<br>Name (official code) | Cumulative no. of tests (as of 26th of March 2020) | Source (official data can be sent on request) |
|---|---|---|
| Dolnośląskie (1 WRO) | 2,360 | Official data |
| Kujawsko-pomorskie (2 BYD) | 1,112 | Official data |
| Lubelskie (3 LUB) | 2,254 | Official data |
| Lubuskie (4 GOR) | 1,953 | Official data |
| Łódzkie (5 LOD) | 3,599 | Official data |
| Małopolskie (6 KRA) | 1,803 | Official data |
| Mazowieckie (7 WAR) | 7,890 | Official data |
| Opolskie (8 OPO) | 488 | Official data |
| Podkarpackie (9 RZE) | 1,591 | Sokalska (2020) |
| Podlaskie (10 BIA) | 867 | Sokalska (2020) |
| Pomorskie (11 GDA) | 1,248 | Official data |
| Śląskie (12 KAT) | 2,672 | Official data |
| Świętokrzyskie (13 KIE) | 1,408 | Official data |
| Warmińsko-mazurskie (14 OLS) | 1,758 | Official data |
| Wielkopolskie (15 POZ) | 1,715 | Official data |
| Zachodniopomorskie (16 SZC) | 759 | Official data |

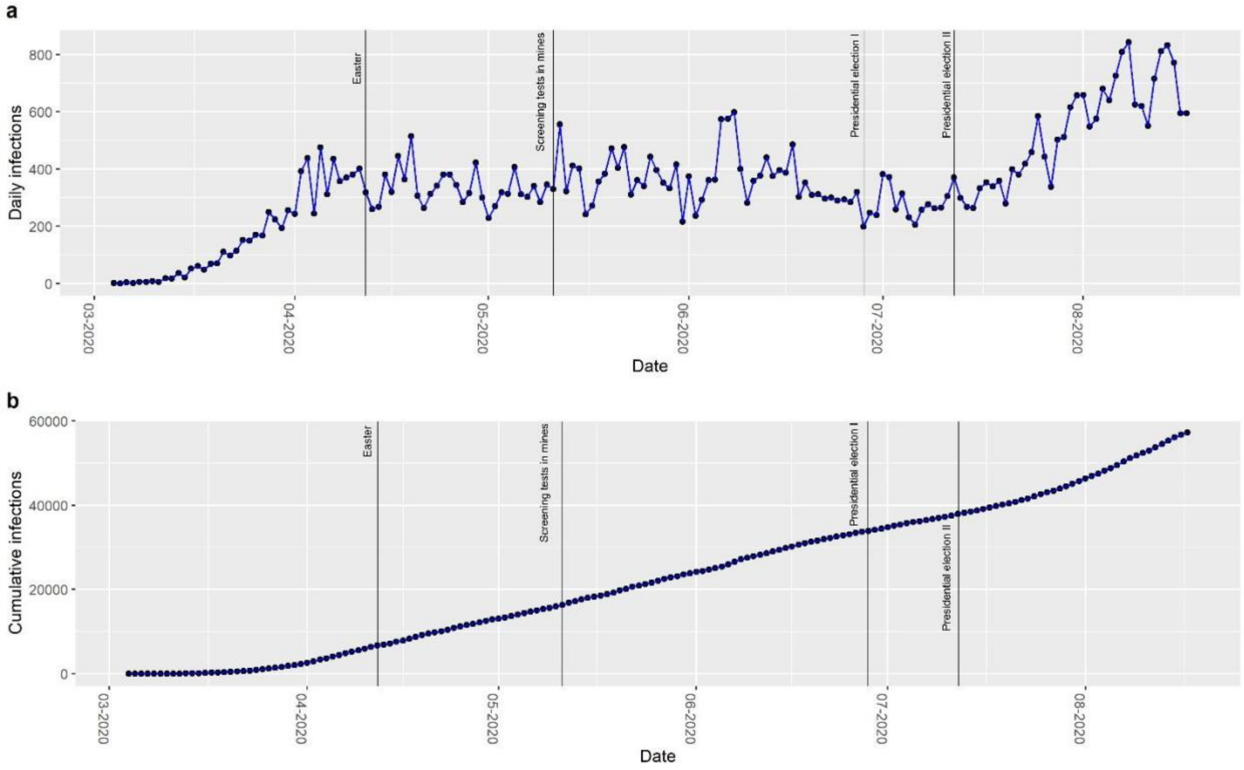


**Figure S1. The development of the epidemic in Poland throughout the study period. A) The number of daily infections; B) the number of cumulative number of infections.**

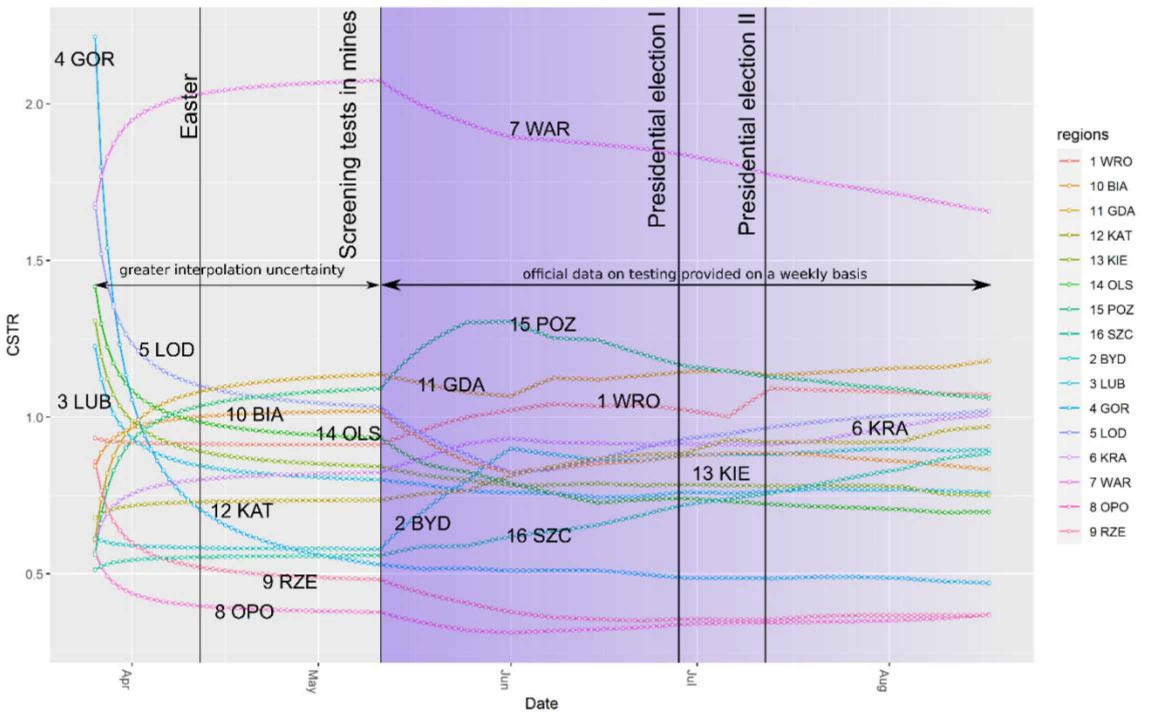


**Figure S2. Relative testing intensity throughout the study period. The data are interpolated which causes 'edge' effects at the nodes.**

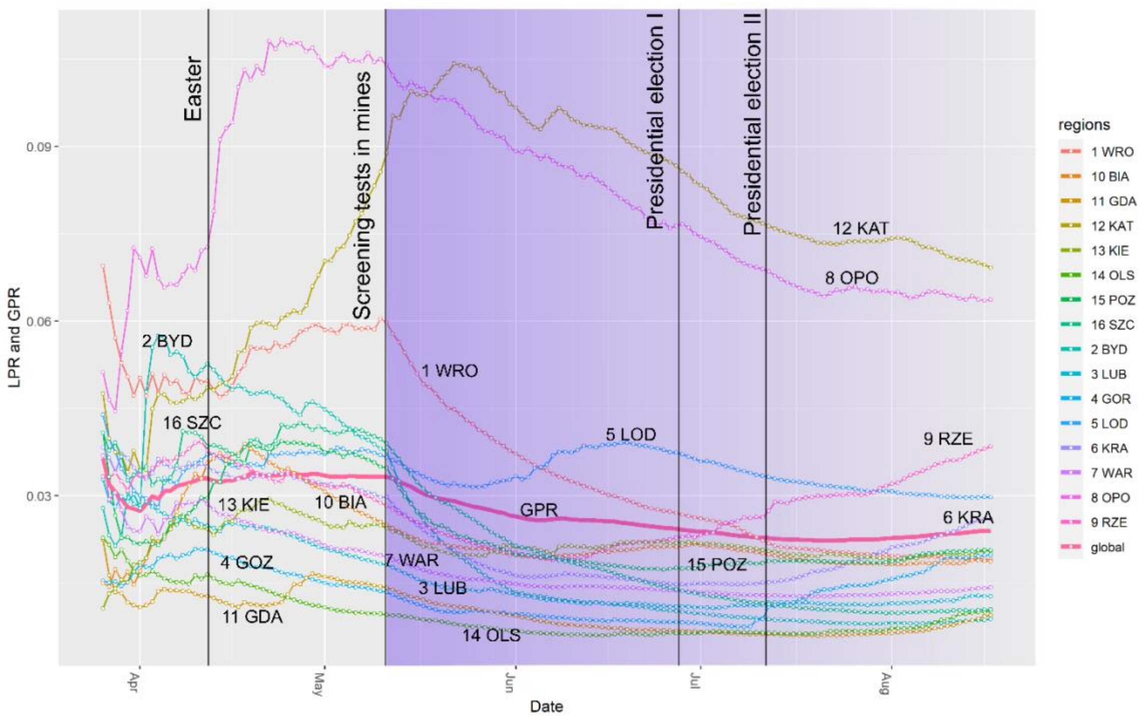


**Figure S3. Local (LPR) and global (GPR) cumulative positivity rates.**

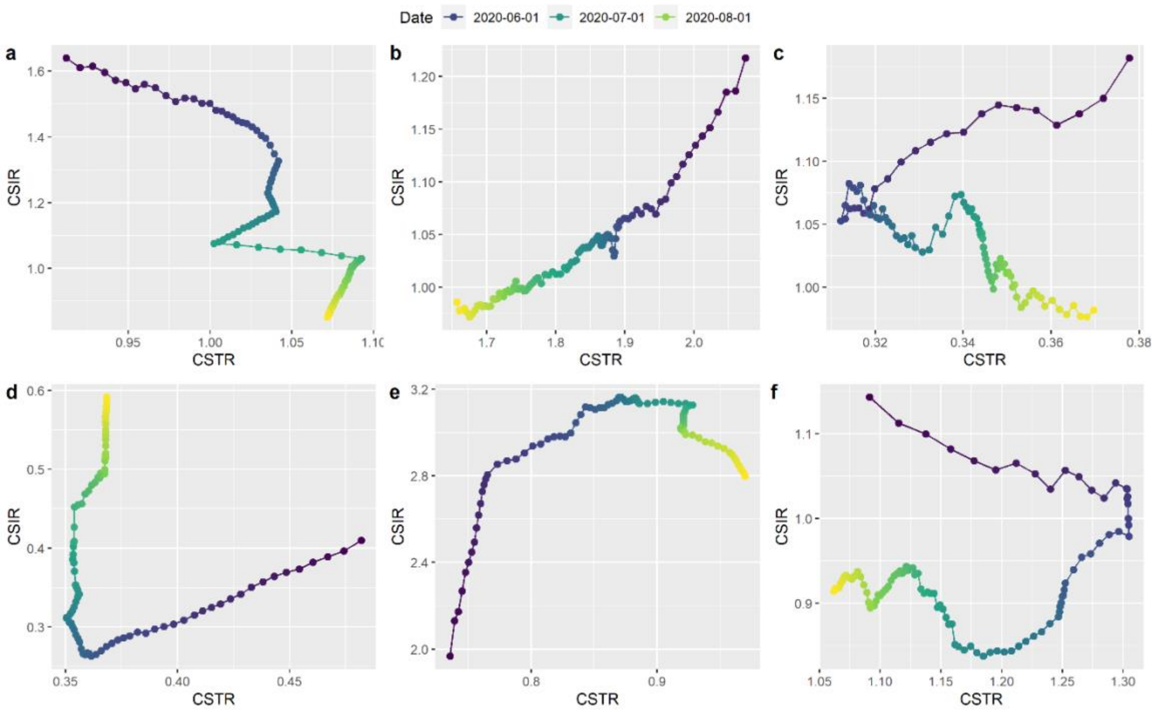


**Figure S4. Relationship between CSTR and CSIR. (A) 1 WRO: CSIR is monotonically decreasing even if the CSTR is sometimes increasing (B) 7 WAR: both CSTR and CSIR are decreasing up to early August, (C) 8 OPO: decreasing CSTR with a simultaneous growth in CSIR in late-May, (D) 9 RZE: CSIR increasing since mid-June, yet CSTR increased in August, (E) 12 KAT: similar growth rate between CSTR and CSIR in early June that followed a greater growth rate of CSIR in May, (F) 15 POZ: decreasing CSTR with undulating CSIR**

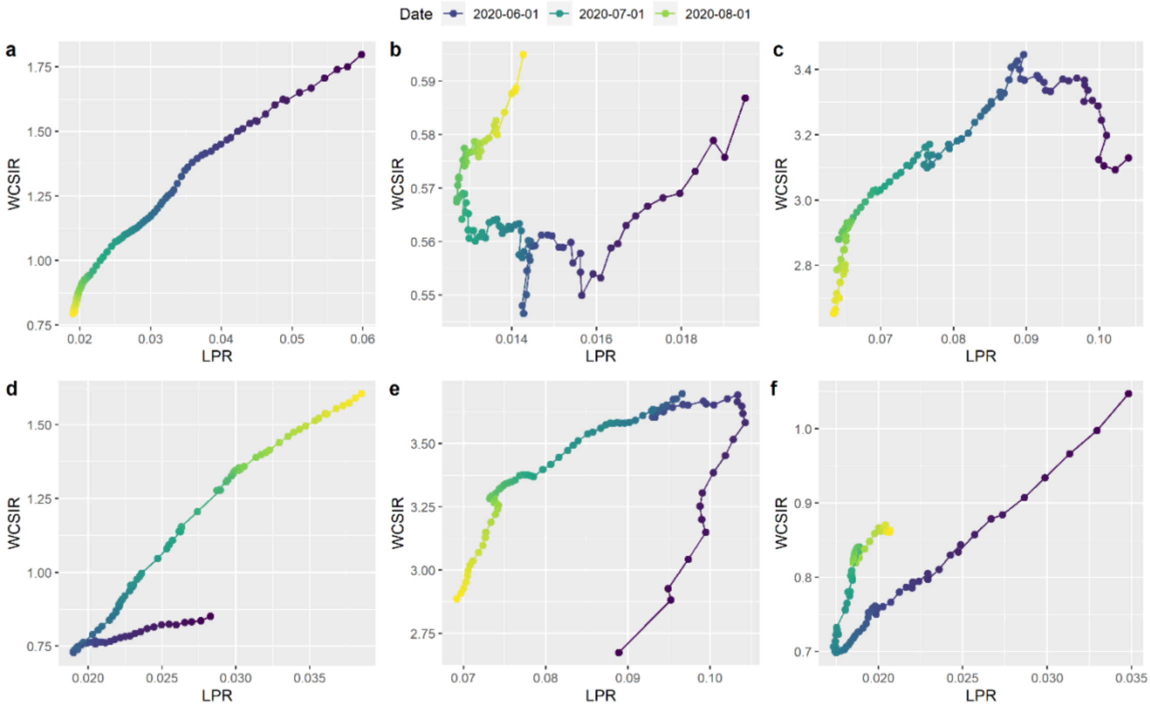


**Figure S5. Relationship between LPR and WCSIR. (A) 1 WRO: simultaneous decrease in LPR and WCSIR (B) 7 WAR: constant LPR with increasing WCSIR in mid-July, (C) 8 OPO: slightly decreasing LPR with an increasing WCSIR in late-May, (D) 9 RZE: both LPR and WCSIR are increasing since June, (E) 12 KAT: LPR and WCSIR are together increasing and then together decreasing, (F) 15 POZ: faster growth of WCSIR than that of LPR since July.**

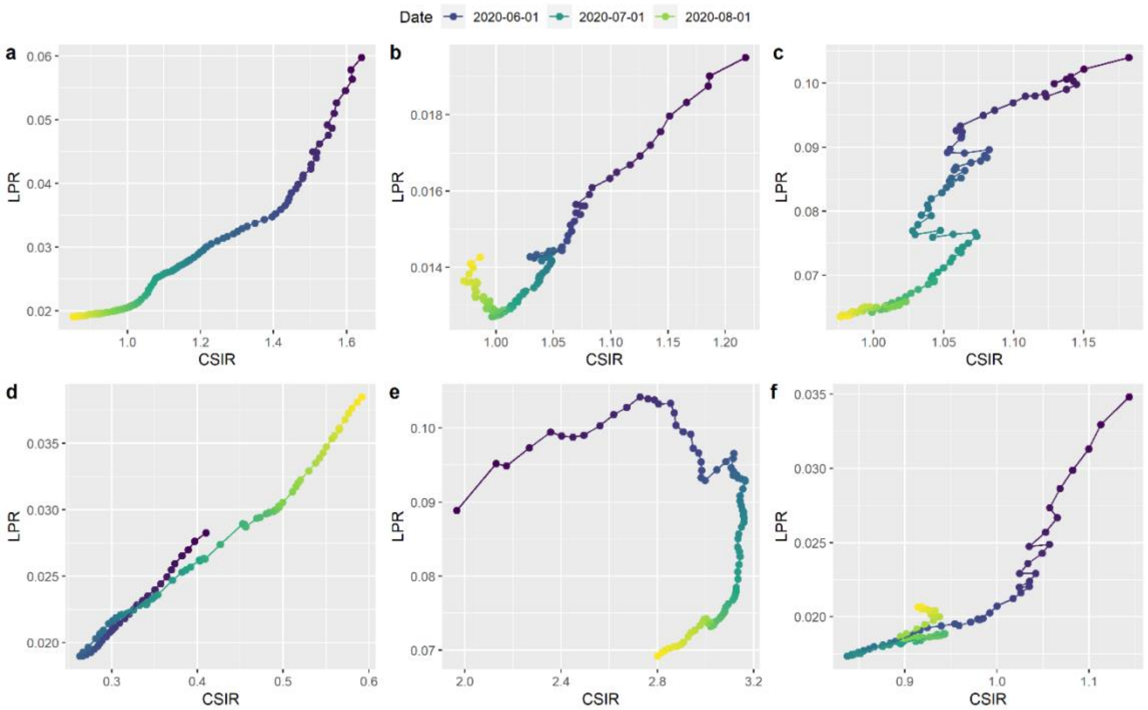


**Figure S6 Relationship between CSIR and LPR. (A) 1 WRO: both CSIR and LPR are decreasing (B) 7 WAR: CSIR is decreasing and LPR is increasing, (C) 8 OPO: a zigzag trajectory, trend shows decreasing CSIR and LPR, (D) 9 RZE: both CSIR and LPR are increasing since mid-June, (E) 12 KAT: constant CSIR with decreasing LPR in July, (F) 15 POZ: change in simultaneous decrease of both CSIR and LPR since July.**

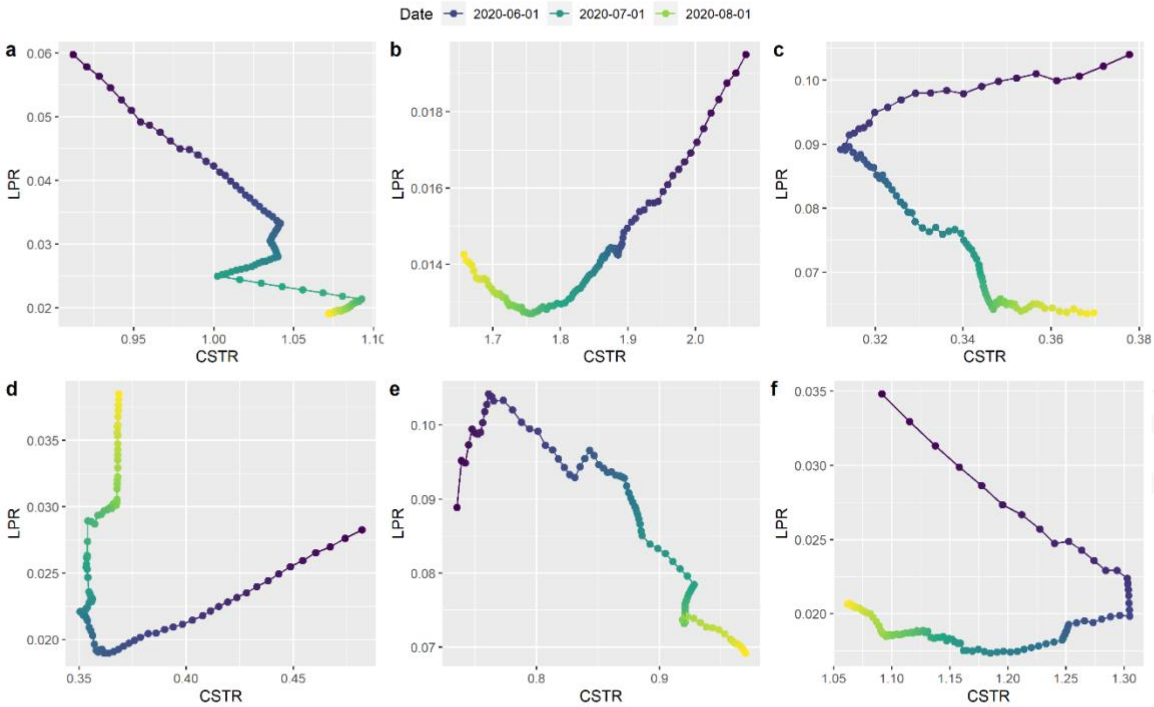


**Figure S7. Relationship between CSTR and LPR. (A) 1 WRO: LPR is monotonically decreasing, CSTR is sometimes increasing (B) 7 WAR: CSTR is decreasing and LPR increasing since late-July, (C) 8 OPO: CSTR increasing since June with a constant LPR in August, (D) 9 RZE: CSTR increased slightly in August with dramatically increasing LPR, (E) 12 KAT: decreasing LPR and increasing CSTR since mid-June, (F) 15 POZ: faster decrease rate for CSTR than the rate of increase of LPR since July.**

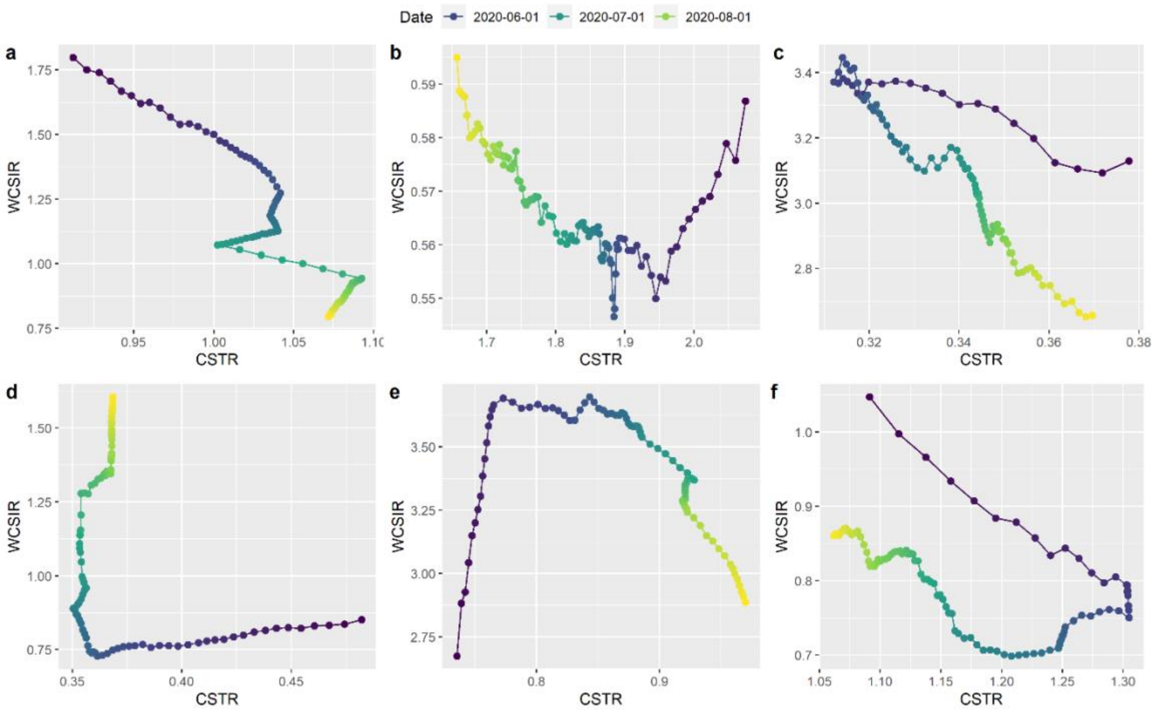


**Figure S8. Relationship between CSTR and WCSIR. (A) 1 WRO: WCSIR is monotonically decreasing, the CSTR is sometimes increasing (B) 7 WAR: CSTR decreasing with a trend of increasing WCSIR since late-May, (C) 8 OPO: decreasing CSTR with a simultaneous growth in WCSIR in mid-May, (D) 9 RZE: CSTR increased slightly in August with a dramatically increasing WCSIR, (E) 12 KAT: decreasing WCSIR since mid-June and increasing CSTR, (F) 15 POZ: decreasing CSTR with undulating WCSIR since July.**

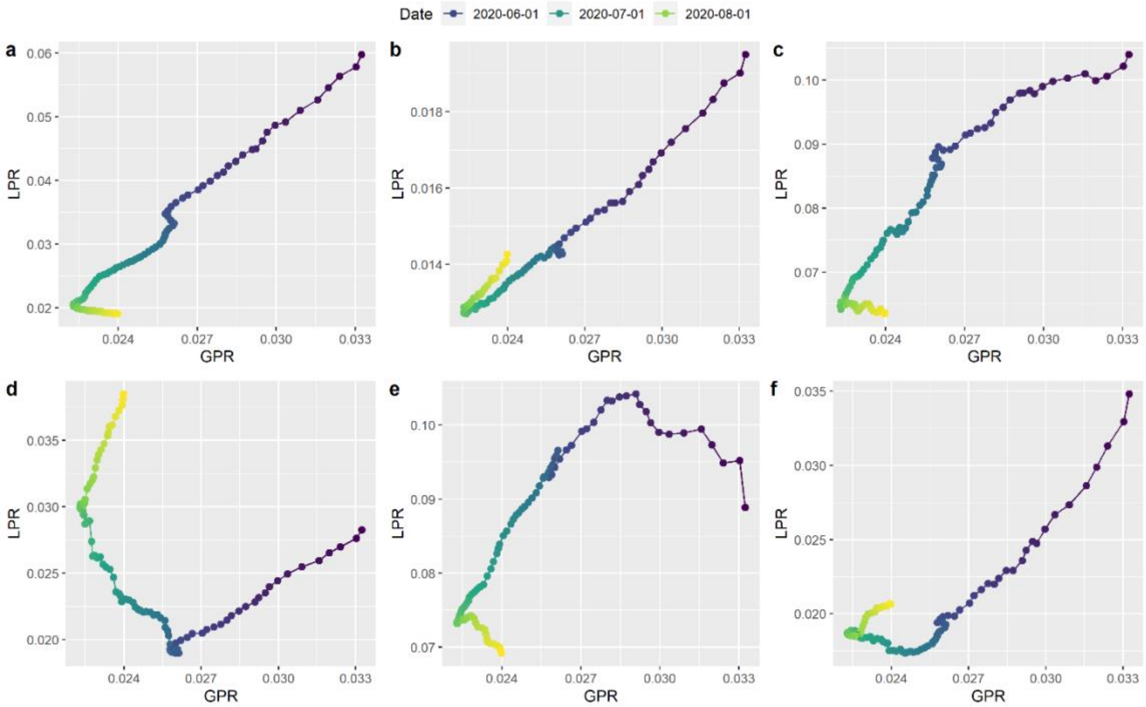


**Figure S9. Relationship between GPR and LPR. (A) 1 WRO: decreasing LPR with increasing GPR in early-June and August, (B) 7 WAR: in August both LPR and GPR are increasing, (C) 8 OPO: in August LPR approximately constant with increasing GPR, (D) 9 RZE: in August LPR increasing faster than GPR, (E) 12 KAT: increasing LPR with decreasing GPR in May, (F) 15 POZ: in August GPR increasing faster than LPR.**

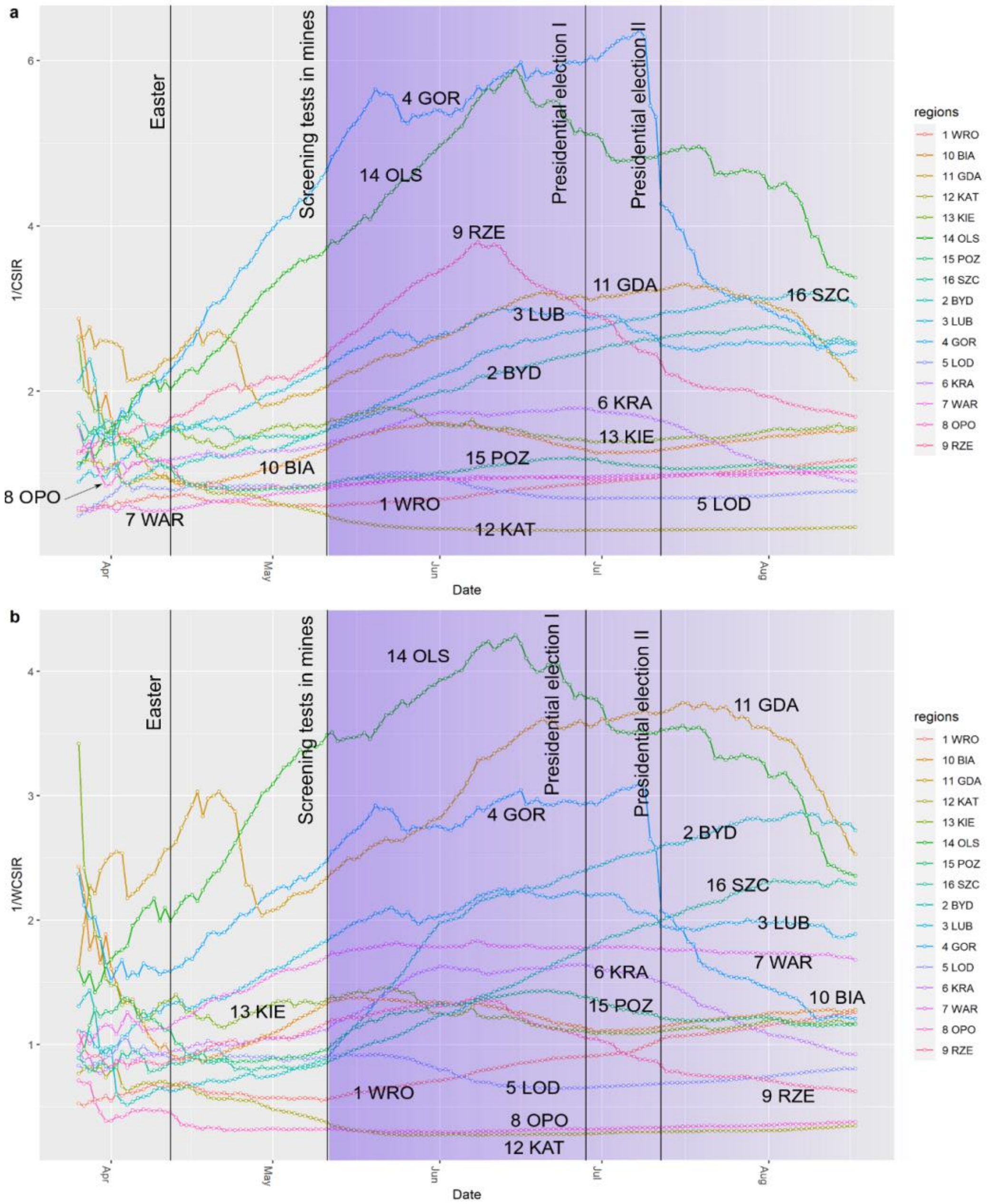


**Figure S10. Relative safety perspective assumed as 1/risk. A) Unweighted safety (1/CSIR); (B) weighted safety (1/WCSIR).**

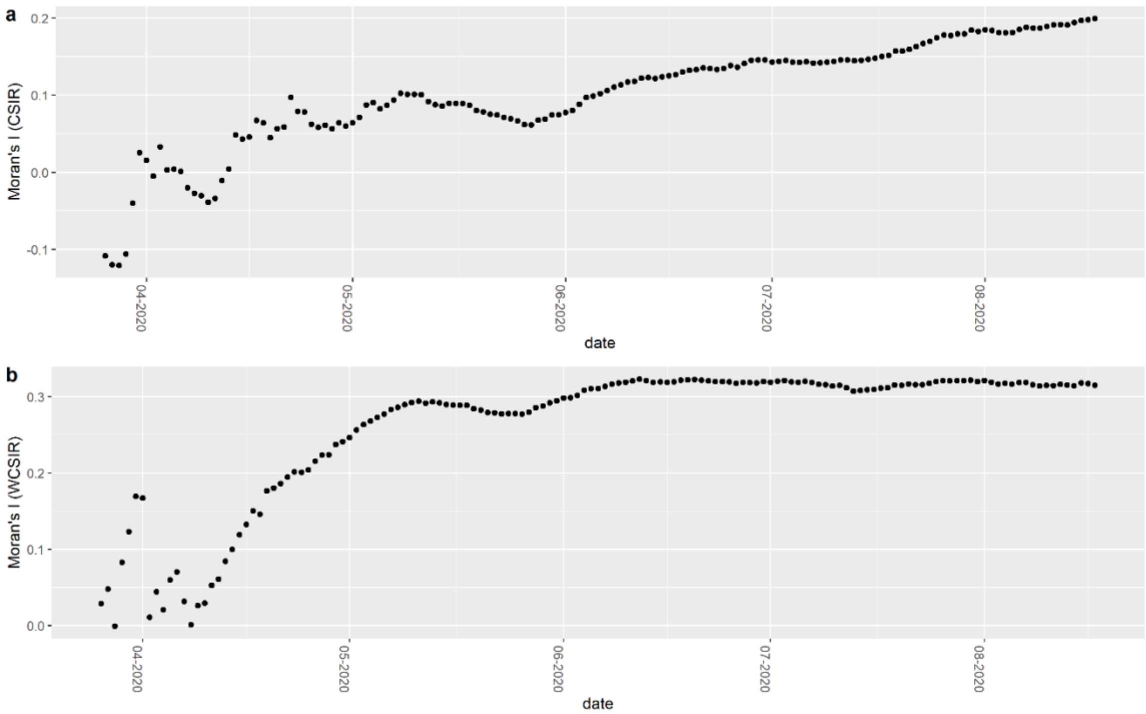


**Figure S11 Assessing spatial autocorrelation using Moran's *I*. A) Biased relative risk (CSIR); (B) unbiased relative risk (WCSIR). It can be seen that the unbiased version indicates greater values of Moran's I, thus better pointing to the synchronisation of epidemics between neighbouring areas.**

## SI References

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